

Improved Stability for Pulsating Multi-Spin String Solitons

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Abstract

We derive and analyse, analytically and numerically, the equations for perturbations around the pulsating two-spin string soliton in $AdS_5 \times S^5$. We show that the pulsation in S^5 indeed improves the stability properties of the two-spin string soliton in AdS_5 , in the sense that the more pulsation we have, the higher spin we can allow and still have stability.

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1 Introduction

The result of semi-classical quantization of long spinning strings in Anti de Sitter space is that the energy E scales with the spin S , $E \sim S$ (for large S). This result, which is independent of the dimensionality of Anti de Sitter space, was originally obtained a decade ago by considering fluctuations around the string center of mass [1]. It holds even classically, as was immediately shown by considering rigidly rotating strings [2], and is due to the constant curvature of Anti de Sitter space. These findings have recently received a lot of attention in connection with the conjectured duality [3, 4, 5] between super string theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SU(N) super Yang-Mills theory in Minkowski space. In the case of rigidly rotating strings, it was noticed [6] that the subleading term is logarithmic in the spin S

$$E - S \sim \ln(S) \tag{1.1}$$

which is essentially the same behavior as found for certain operators on the gauge theory side [7, 8, 9, 10, 11]. The E - S relationship has been further investigated in a number of papers including [12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29] for various string configurations.

More recently it was discovered that multi spin solutions could also be constructed easily [30]. Among others, a relatively simple solution was found in AdS_5 describing a string which is located at a point in the radial direction, winding around an angular direction and spinning with equal angular momentum in two independent planes. In the long string limit it was shown that

$$E - 2S \sim S^{1/3} \tag{1.2}$$

Multi spin solitons in $AdS_5 \times S^5$ have been further investigated in a number of papers including [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52]. However, most of these papers have dealt with multi spin solutions in the S^5 part of $AdS_5 \times S^5$.

The main problem with all these multi spin solutions, is that they tend

to be classically unstable for large spin. For instance, the simple two-spin solution leading to (1.2) is only stable for [30]

$$S \leq \frac{5\sqrt{7}}{8\sqrt{2}H^2\alpha'} \quad (1.3)$$

This means, strictly speaking, that the result (1.2) can not be trusted, since it was derived under the assumption that $S \gg (H^2\alpha')^{-1}$.

The purpose of the present paper is to show that pulsation improves the stability properties for multi spin string solitons. More precisely, we take the simple two-spin string solution in AdS_5 [30], couple it with pulsation in S^5 and consider small perturbations around it. We show that this solution has better stability properties than the non-pulsating one.

The paper is organised as follows. In section 2, we give a short review of the pulsating two-spin solution in $AdS_5 \times S^5$ [26]. Section 3 is devoted to a general discussion of perturbations, and in section 4 we derive the equations of motion for the physical perturbations around the pulsating two-spin solution. In section 5, we solve the equations analytically for a few tractable cases, while leaving numerical analysis of the general case for section 6. Finally, in section 7, we present our conclusions.

2 Pulsating Multi Spin Solutions

In this section, we set our conventions and notations, and review the pulsating two-spin string soliton [26].

We have altogether for $AdS_5 \times S^5$, the line-element

$$\begin{aligned} ds^2 = & -(1 + H^2 r^2) dt^2 + \frac{dr^2}{1 + H^2 r^2} + r^2 (d\beta^2 + \sin^2 \beta d\phi^2 + \cos^2 \beta d\tilde{\phi}^2) \\ & + \frac{1}{H^2} (d\theta^2 + \sin^2 \theta d\psi^2 + \cos^2 \theta (d\psi_1^2 + \sin^2 \psi_1 d\psi_2^2 + \cos^2 \psi_1 d\psi_3^2)) \end{aligned} \quad (2.1)$$

The 't Hooft coupling in this notation is $\lambda = (H^2\alpha')^{-2}$, where $(2\pi\alpha')^{-1}$ is the string tension. The radius of S^5 is H^{-1} .

In the standard parametrisation of S^3 we have the ranges $\beta \in [0, \pi/2]$, $\phi \in [0, 2\pi]$, $\tilde{\phi} \in [0, 2\pi]$, but here we shall use the alternative ranges $\beta \in [0, 2\pi]$, $\phi \in [0, \pi]$, $\tilde{\phi} \in [0, \pi]$.

We take the Polyakov action

$$S = -\frac{1}{4\pi\alpha'} \int d\tau d\sigma \sqrt{-h} h^{\alpha\beta} G_{\mu\nu} X_{,\alpha}^\mu X_{,\beta}^\nu \quad (2.2)$$

using the conformal gauge

$$G_{\mu\nu} \dot{X}^\mu X'^\nu = 0, \quad G_{\mu\nu} (\dot{X}^\mu \dot{X}^\nu + X'^\mu X'^\nu) = 0 \quad (2.3)$$

where dot and prime denote derivatives with respect to the world-sheet coordinates τ and σ , such that the equations of motion are

$$\ddot{X}^\mu - X''^\mu + \Gamma_{\rho\sigma}^\mu (\dot{X}^\rho \dot{X}^\sigma - X'^\rho X'^\sigma) = 0 \quad (2.4)$$

The ansatz for the pulsating two-spin string soliton is

$$\begin{aligned} t &= c_0 \tau, & r &= r_0, & \beta &= \sigma, & \phi &= \omega \tau, & \tilde{\phi} &= \omega \tau, \\ \theta &= \theta(\tau), & \psi &= \sigma, & \psi_1 &= \psi_{10}, & \psi_2 &= \psi_{20}, & \psi_3 &= \psi_{30} \end{aligned} \quad (2.5)$$

where $(c_0, r_0, \omega, \psi_{10}, \psi_{20}, \psi_{30})$ are arbitrary constants. This is a circular string in AdS_5 , spinning in two different directions. It is also a circle in S^5 , but pulsating there. The r and θ equations become

$$\omega^2 = 1 + H^2 c_0^2 \quad (2.6)$$

$$\ddot{\theta} + \sin \theta \cos \theta = 0 \quad (2.7)$$

while the non-trivial conformal gauge constraint is

$$\dot{\theta}^2 + \sin^2 \theta - H^2 (c_0^2 - 2r_0^2) = 0 \quad (2.8)$$

The θ equation and constraint are solved by

$$\sin \theta(\tau) = \begin{cases} A \operatorname{sn}(\tau|A^2), & A \leq 1 \\ \operatorname{sn}(A\tau|1/A^2), & A \geq 1 \end{cases} \quad (2.9)$$

where

$$A = H\sqrt{c_0^2 - 2r_0^2} \quad (2.10)$$

Notice that (2.7) is equivalent to the equation for a simple plane pendulum. It means that we have two types of motion, and a limiting case. More precisely, for $A < 1$ the string oscillates on one hemisphere, for $A = 1$ it starts at one of the poles and approaches the equator for $\tau \rightarrow \infty$, while for $A > 1$ it oscillates between the poles.

The energy E and the 2 spins $S_1 = S_2 \equiv S$ are easily computed

$$E = \frac{(1 + H^2 r_0^2) \sqrt{2H^2 r_0^2 + A^2}}{H\alpha'} \quad (2.11)$$

$$S = \frac{r_0^2}{2\alpha'} \sqrt{2r_0^2 H^2 + A^2 + 1} \quad (2.12)$$

For short strings (say $Hr_0 \ll 1$) we get

$$S \approx \frac{r_0^2}{2\alpha'} \sqrt{A^2 + 1} \left(1 + \frac{H^2 r_0^2}{A^2 + 1} \right) \quad (2.13)$$

such that

$$H^2 r_0^2 \approx \frac{2H^2 \alpha' S}{\sqrt{A^2 + 1}} - \frac{4H^4 \alpha'^2 S^2}{(A^2 + 1)^2} \quad (2.14)$$

which inserted into E gives

$$E(S, A) \approx \frac{1}{H\alpha'} \left(1 + \frac{2H^2 \alpha' S}{\sqrt{A^2 + 1}} - \frac{4H^4 \alpha'^2 S^2}{(A^2 + 1)^2} \right) \sqrt{\frac{4H^2 \alpha' S}{\sqrt{A^2 + 1}} - \frac{8H^4 \alpha'^2 S^2}{(A^2 + 1)^2} + A^2} \quad (2.15)$$

For $A = 0$ we get

$$E \approx \frac{2\sqrt{S}}{\sqrt{\alpha'}} (1 + H^2 \alpha' S) \quad (2.16)$$

which to leading order is just the Minkowski result $\alpha' E^2 = 2(2S)$. For $A = 1$ we get

$$E \approx \frac{1}{H\alpha'} (1 + 2^{3/2} H^2 \alpha' S) \quad (2.17)$$

and for $A \gg 1$

$$E \approx \frac{1}{H\alpha'}(A + 2H^2\alpha'S) \quad (2.18)$$

For long strings (say $Hr_0 \gg 1$) we get

$$E/H - 2S \approx \frac{\sqrt{2H^2r_0^2 + A^2}}{H^2\alpha'} - \frac{H^2r_0^2}{2H^2\alpha'\sqrt{2H^2r_0^2 + A^2}} \quad (2.19)$$

Now we have to distinguish between different cases. If $Hr_0 \gg A$, we get from (2.12)

$$S \approx \frac{Hr_0^3}{\sqrt{2}\alpha'} \left(1 + \frac{A^2 + 1}{4H^2r_0^2} \right) \quad (2.20)$$

such that

$$Hr_0 \approx (\sqrt{2}H^2\alpha'S)^{1/3} - \frac{A^2 + 1}{12(\sqrt{2}H^2\alpha'S)^{1/3}} \quad (2.21)$$

which, when inserted into (2.19), gives

$$E/H - 2S \approx \frac{3(\sqrt{2}H^2\alpha'S)^{1/3}}{2^{3/2}H^2\alpha'} + \frac{4A^2 - 1}{2^{7/2}H^2\alpha'(\sqrt{2}H^2\alpha'S)^{1/3}} \quad (2.22)$$

This result is valid for $H^2\alpha'S \gg \{1, A^3\}$, and therefore holds in particular for $A = 0$ and $A = 1$. Notice also that the pulsation only gives a contribution to the non-leading terms, in this limit. On the other hand, if $Hr_0 \ll A$ we get from eq.(2.12)

$$S \approx \frac{Ar_0^2}{2\alpha'} \left(1 + \frac{r_0^2H^2}{A^2} \right) \quad (2.23)$$

such that

$$H^2r_0^2 \approx \frac{2H^2S\alpha'}{A} - \frac{4S^2\alpha'^2H^4}{A^4} \quad (2.24)$$

and insertion into (2.19) gives

$$E/H - 2S \approx \frac{A}{H^2\alpha'} + \frac{S}{A^2} \quad (2.25)$$

which holds for $1 \ll H^2\alpha'S \ll A^3$. Thus, in this limit, the pulsation completely changes the scaling relation.

For $A = 0$ all the results of this section reduce to those obtained in [30], but are otherwise quite different. The main problem is that the string soliton is generally not stable. For $A = 0$ it was shown in [30] that there is only stability for

$$S \leq \frac{5\sqrt{7}}{8\sqrt{2}H^2\alpha'} \quad (2.26)$$

The purpose of the following sections is to find the condition for stability for arbitrary A .

3 Perturbations

There are many ways to discuss the perturbations around string solitons. One way is to use the Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det(G_{\mu\nu}X_{,\alpha}^{\mu}X_{,\beta}^{\nu})} \quad (3.1)$$

and make variations δX^{μ} to obtain $\delta^2 S$, immediately fixing two of them so as to take care of the gauge invariance. This is the most often used method in recent papers. The disadvantages are that it is not world-sheet covariant, and that the kinetic energy terms for the perturbations usually come out in a very complicated form.

Another way is to take the Polyakov action in conformal gauge; i.e., one makes a variation of (2.3) and two variations of (see [53, 54, 55])

$$S = \frac{1}{4\pi\alpha'} \int d\tau d\sigma G_{\mu\nu}(\dot{X}^{\mu}\dot{X}^{\nu} - X'^{\mu}X'^{\nu}) \quad (3.2)$$

This approach is also used in many papers, but the problem here is that one can not usually solve the variation of (2.3), making it is very difficult to separate the physical and unphysical perturbations.

Here we will use the Polyakov action (2.2); i.e., make two variations δX^μ and $\delta h_{\alpha\beta}$. To ensure that we have purely physical perturbations, we shall consider only perturbations which are normal to the string world-sheet. This approach is in general world-sheet covariant, and we will eventually get the kinetic energy terms in a very simple form (see [56, 57, 58, 59]).

First introduce 8 normal vectors to the world-sheet ($i = 1, 2, \dots, 8$) fulfilling

$$G_{\mu\nu} N_i^\mu N_j^\nu = \delta_{ij}, \quad G_{\mu\nu} N_i^\mu X_{,\alpha}^\nu = 0 \quad (3.3)$$

as well as the completeness relation

$$G^{\mu\nu} = \frac{2h^{\alpha\beta}}{h^{\gamma\delta} g_{\gamma\delta}} X_{,\alpha}^\mu X_{,\beta}^\nu + \delta^{ij} N_i^\mu N_j^\nu \quad (3.4)$$

where $g_{\alpha\beta}$ is the induced metric on the world-sheet. Then define the second fundamental form and normal fundamental form

$$K_{\alpha\beta}^i = N_\mu^i X_{,\beta}^\rho \nabla_\rho X_{,\alpha}^\mu, \quad \mu_\alpha^{ij} = N_\mu^i X_{,\alpha}^\rho \nabla_\rho N^{j\mu} \quad (3.5)$$

where ∇_ρ is the covariant derivative with respect to the metric $G_{\mu\nu}$.

Since $(X_{,\alpha}^\mu, N_i^\mu)$ is a basis for spacetime (at least locally), we can write

$$\delta X^\mu = N_i^\mu \phi^i + X_{,\alpha}^\mu \psi^\alpha \quad (3.6)$$

The ψ^α are just reparametrisations, and it can then be shown that the ϕ^i fulfil the equations (see [56, 57, 58, 59])

$$\left(\delta^{kl} h^{\alpha\beta} D_{ik\alpha} D_{lj\beta} + \frac{2}{h^{\epsilon\zeta} g_{\epsilon\zeta}} K_i^{\alpha\beta} K_{j\alpha\beta} + h^{\alpha\beta} R_{\mu\rho\nu\sigma} N_i^\mu N_j^\nu X_{,\alpha}^\rho X_{,\beta}^\sigma \right) \phi^j = 0 \quad (3.7)$$

where we introduced $D_{ij\alpha} = \delta_{ij} D_\alpha + \mu_{ij\alpha}$, and D_α is the covariant derivative with respect to the Polyakov metric.

4 Stability of Pulsating Two-Spin String Soliton

As the unperturbed string we take the pulsating two-spin solution in $AdS_5 \times S^5$ from section 2. That is, $h_{\alpha\beta} = \eta_{\alpha\beta} = \text{diag}(-1, 1)$ as well as

$$X^\mu = (c_0\tau, r_0, \sigma, \omega\tau, \omega\tau, \theta(\tau), \sigma, \psi_{10}, \psi_{20}, \psi_{30}) \quad (4.1)$$

The tangent vectors are

$$\dot{X}^\mu = (c_0, 0, 0, \omega, \omega, \dot{\theta}, 0, 0, 0, 0) \quad (4.2)$$

$$X'^\mu = (0, 0, 1, 0, 0, 0, 1, 0, 0, 0) \quad (4.3)$$

The induced metric on the world-sheet is

$$g_{\tau\sigma} = 0, \quad g_{\tau\tau} = -g_{\sigma\sigma} = -H^{-2}(H^2r_0^2 + \sin^2\theta) \quad (4.4)$$

There are 8 normal vectors N_μ^i fulfilling eqs.(3.3)-(3.4). They can be chosen as

$$N_\mu^1 = \frac{r_0\sqrt{1+H^2r_0^2}}{\sqrt{c_0^2-r_0^2}}(\omega, 0, 0, -c_0\sin^2\sigma, -c_0\cos^2\sigma, 0, 0, 0, 0, 0) \quad (4.5)$$

$$N_\mu^2 = r_0\sin\sigma\cos\sigma(0, 0, 0, 1, -1, 0, 0, 0, 0, 0) \quad (4.6)$$

$$N_\mu^3 = (1+H^2r_0^2)^{-1/2}(0, 1, 0, 0, 0, 0, 0, 0, 0, 0) \quad (4.7)$$

$$N_\mu^4 = \frac{r_0\sin\theta}{\sqrt{H^2r_0^2+\sin^2\theta}}(0, 0, 1, 0, 0, 0, -1, 0, 0, 0) \quad (4.8)$$

$$N_\mu^5 = \frac{1}{\sqrt{c_0^2-r_0^2}\sqrt{H^2r_0^2+\sin^2\theta}} \left(-c_0(1+H^2r_0^2)\dot{\theta}, 0, 0, \omega r_0^2\sin^2\sigma\dot{\theta}, \omega r_0^2\cos^2\sigma\dot{\theta}, c_0^2-r_0^2, 0, 0, 0, 0 \right) \quad (4.9)$$

$$N_\mu^6 = H^{-1}\cos\theta(0, 0, 0, 0, 0, 0, 1, 0, 0, 0) \quad (4.10)$$

$$N_\mu^7 = H^{-1}\cos\theta\sin\psi_{10}(0, 0, 0, 0, 0, 0, 0, 1, 0, 0) \quad (4.11)$$

$$N_\mu^8 = H^{-1}\cos\theta\cos\psi_{10}(0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \quad (4.12)$$

Notice that the first 3 only have components in AdS_5 , while the last 3 only have components in S^5 . It is then straightforward to compute the second fundamental form, the normal fundamental form and the projections of the Riemann tensor, appearing in (3.7). For convenience, they are listed in the appendix. We now have all the ingredients to write down the equations (3.7) for the perturbations

$$-\ddot{\phi}^1 + \phi''^1 + \frac{2c_0\omega}{\sqrt{c_0^2 - r_0^2}} \dot{\phi}^3 + \frac{2c_0\sqrt{1 + H^2 r_0^2}}{\sqrt{c_0^2 - r_0^2}} \phi'^2 = 0 \quad (4.13)$$

$$\begin{aligned} & -\ddot{\phi}^2 + \phi''^2 - \frac{2\omega \sin \theta}{\sqrt{H^2 r_0^2 + \sin^2 \theta}} \dot{\phi}^4 - \frac{2c_0\sqrt{1 + H^2 r_0^2}}{\sqrt{c_0^2 - r_0^2}} \phi'^1 \\ & + \frac{2r_0\omega\dot{\theta}}{\sqrt{c_0^2 - r_0^2}\sqrt{H^2 r_0^2 + \sin^2 \theta}} \phi'^5 - \frac{4H^2 r_0^2(H^2 c_0^2 + 1)}{H^2 r_0^2 + \sin^2 \theta} \phi^2 \\ & + \frac{2H^2 r_0^2 \omega \cos \theta \dot{\theta}}{(H^2 r_0^2 + \sin^2 \theta)^{3/2}} \phi^4 = 0 \end{aligned} \quad (4.14)$$

$$\begin{aligned} & -\ddot{\phi}^3 + \phi''^3 - \frac{2c_0\omega}{\sqrt{c_0^2 - r_0^2}} \dot{\phi}^1 + \frac{2r_0\dot{\theta}\sqrt{1 + H^2 r_0^2}}{\sqrt{c_0^2 - r_0^2}\sqrt{H^2 r_0^2 + \sin^2 \theta}} \dot{\phi}^5 \\ & - \frac{2\sqrt{1 + H^2 r_0^2} \sin \theta}{\sqrt{H^2 r_0^2 + \sin^2 \theta}} \phi'^4 + \frac{4H^2 r_0^2(1 + H^2 r_0^2)}{H^2 r_0^2 + \sin^2 \theta} \phi^3 \\ & + \frac{2H^2 r_0 \cos \theta \sin \theta \sqrt{1 + H^2 r_0^2} \sqrt{c_0^2 - r_0^2}}{(H^2 r_0^2 + \sin^2 \theta)^{3/2}} \phi^5 = 0 \end{aligned} \quad (4.15)$$

$$\begin{aligned} & -\ddot{\phi}^4 + \phi''^4 + \frac{2\omega \sin \theta}{\sqrt{H^2 r_0^2 + \sin^2 \theta}} \dot{\phi}^2 + \frac{2\sqrt{1 + H^2 r_0^2} \sin \theta}{\sqrt{H^2 r_0^2 + \sin^2 \theta}} \phi'^3 \\ & - \frac{2H^2 r_0 \cos \theta \sqrt{c_0^2 - r_0^2}}{H^2 r_0^2 + \sin^2 \theta} \phi'^5 + \frac{4H^2 r_0^2 \omega \dot{\theta} \cos \theta}{(H^2 r_0^2 + \sin^2 \theta)^{3/2}} \phi^2 \\ & + \frac{H^2 r_0^2 [-3H^2 c_0^2 + 5H^2 r_0^2 + 2H^4 r_0^4 - H^4 c_0^2 r_0^2]}{(H^2 r_0^2 + \sin^2 \theta)^2} \phi^4 \\ & + \frac{H^2 r_0^2 [(2 - 2H^2 r_0^2 + 2H^2 c_0^2) \sin^2 \theta - \sin^4 \theta]}{(H^2 r_0^2 + \sin^2 \theta)^2} \phi^4 = 0 \end{aligned} \quad (4.16)$$

$$\begin{aligned}
& -\ddot{\phi}^5 + \phi''^5 - \frac{2r_0\dot{\theta}\sqrt{1+H^2r_0^2}}{\sqrt{c_0^2-r_0^2}\sqrt{H^2r_0^2+\sin^2\theta}}\dot{\phi}^3 \\
& - \frac{2r_0\omega\dot{\theta}}{\sqrt{c_0^2-r_0^2}\sqrt{H^2r_0^2+\sin^2\theta}}\phi'^2 + \frac{2H^2r_0\cos\theta\sqrt{c_0^2-r_0^2}}{H^2r_0^2+\sin^2\theta}\phi'^4 \\
& + \frac{4H^2r_0\sin\theta\cos\theta\sqrt{c_0^2-r_0^2}\sqrt{1+H^2r_0^2}}{(H^2r_0^2+\sin^2\theta)^{3/2}}\phi^3 \\
& + \frac{H^2(c_0^2-r_0^2)(-H^2r_0^2+2(H^2r_0^2+1)\sin^2\theta-\sin^4\theta)}{(H^2r_0^2+\sin^2\theta)^2}\phi^5 = 0 \quad (4.17)
\end{aligned}$$

$$-\ddot{\phi}^6 + \phi''^6 + (2\sin^2\theta - H^2(c_0^2 - 2r_0^2))\phi^6 = 0 \quad (4.18)$$

$$-\ddot{\phi}^7 + \phi''^7 + (2\sin^2\theta - H^2(c_0^2 - 2r_0^2))\phi^7 = 0 \quad (4.19)$$

$$-\ddot{\phi}^8 + \phi''^8 + (2\sin^2\theta - H^2(c_0^2 - 2r_0^2))\phi^8 = 0 \quad (4.20)$$

To solve eqs.(4.13)-(4.20), we make Fourier expansions

$$\phi^i(\tau, \sigma) = \sum_n e^{in\sigma} \phi_n^i(\tau) \quad (4.21)$$

where $\phi_n^{i*} = \phi_{-n}^i$. We also insert the explicit solution (2.9) and define $\kappa = Hc_0$. The equations are now parametrised by the two dimensionless parameters (A, κ) , where according to eq.(2.10) $\kappa \geq A$.

Notice that we have 5 coupled equations for $\phi_n^1 - \phi_n^5$, and 3 decoupled equations for the others. Let us first see what we can say about the 3 identical decoupled equations. They are given by (we skip the i index)

$$\ddot{\phi}_n + \left(A^2 + n^2 - 2A^2 \text{sn}^2(\tau|A^2) \right) \phi_n = 0, \quad A \leq 1 \quad (4.22)$$

$$\ddot{\phi}_n + \left(A^2 + n^2 - 2\text{sn}^2(A\tau|1/A^2) \right) \phi_n = 0, \quad A \geq 1 \quad (4.23)$$

For $n \neq 0$, the factor in front of ϕ_n is periodic and positive, making the solution stable. The $n = 0$ mode is just a zero-mode redefining the unperturbed solution.

Thus we are left with the first 5 equations.

5 Analytical Results

First we look at the equations in some special limits. For $A = 0$, corresponding to no pulsation, we get

$$\ddot{\phi}_n^1 + n^2 \phi_n^1 - \sqrt{8(1 + \kappa^2)} \dot{\phi}_n^3 - \sqrt{4(2 + \kappa^2)} in \phi_n^2 = 0 \quad (5.1)$$

$$\ddot{\phi}_n^2 + n^2 \phi_n^2 + \sqrt{4(2 + \kappa^2)} in \phi_n^1 + 4(1 + \kappa^2) \phi_n^2 = 0 \quad (5.2)$$

$$\ddot{\phi}_n^3 + n^2 \phi_n^3 + \sqrt{8(1 + \kappa^2)} \dot{\phi}_n^1 - 2(2 + \kappa^2) \phi_n^3 = 0 \quad (5.3)$$

$$\ddot{\phi}_n^4 + n^2 \phi_n^4 + 2in \phi_n^5 + \phi_n^4 = 0 \quad (5.4)$$

$$\ddot{\phi}_n^5 + n^2 \phi_n^5 - 2in \phi_n^4 + \phi_n^5 = 0 \quad (5.5)$$

The first 3 equations are of course the same as in [30], and are stable for $\kappa^2 \leq 5/2$. We are then left with the last two equations which can be written (we define $p_n^4 = \dot{\phi}_n^4$ and $p_n^5 = \dot{\phi}_n^5$)

$$\begin{pmatrix} \dot{\phi}_n^4 \\ \dot{p}_n^4 \\ \dot{\phi}_n^5 \\ \dot{p}_n^5 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 + n^2 & 0 & 2in & 0 \\ 0 & 0 & 0 & -1 \\ -2in & 0 & 1 + n^2 & 0 \end{pmatrix} \begin{pmatrix} \phi_n^4 \\ p_n^4 \\ \phi_n^5 \\ p_n^5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (5.6)$$

To have stability we must have that all eigenvalues of the matrix are imaginary. The eigenvalues are

$$\begin{aligned} \lambda_+ &= \pm i|1 - n| \\ \lambda_- &= \pm i|1 + n| \end{aligned} \quad (5.7)$$

meaning stability. The modes corresponding to $n = \pm 1$ are zero-modes. In conclusion, for $A = 0$ we have stability for $\kappa^2 \leq 5/2$, corresponding to spin (2.12)

$$S \leq \frac{5\sqrt{7}}{8\sqrt{2}H^2\alpha'} \quad (5.8)$$

in agreement with [30].

Another special case of importance is $A = 1$, corresponding to a string which starts at one of the poles and approaches the equator for $\tau \rightarrow \infty$. Unfortunately, the equations do not really simplify in this limit. However, if we only look at the asymptotic equations, we get

$$\ddot{\phi}_n^1 + n^2 \phi_n^1 - 2(\kappa i n \phi_n^2 + \sqrt{2} \kappa \dot{\phi}_n^3) = 0 \quad (5.9)$$

$$\ddot{\phi}_n^2 + n^2 \phi_n^2 + 2(\kappa i n \phi_n^1 + \sqrt{2} \dot{\phi}_n^4) + 4(\kappa^2 - 1) \phi_n^2 = 0 \quad (5.10)$$

$$\ddot{\phi}_n^3 + n^2 \phi_n^3 + 2(i n \phi_n^4 + \sqrt{2} \kappa \dot{\phi}_n^1) - 2(\kappa^2 - 1) \phi_n^3 = 0 \quad (5.11)$$

$$\ddot{\phi}_n^4 + n^2 \phi_n^4 - 2(i n \phi_n^3 + \sqrt{2} \dot{\phi}_n^2) = 0 \quad (5.12)$$

$$\ddot{\phi}_n^5 + (n^2 - 1) \phi_n^5 = 0 \quad (5.13)$$

It is of course somewhat dangerous to take $\tau \rightarrow \infty$ in the equations, since it corresponds to setting a simple plane pendulum in the vertical upright position, c.f. the comments after eq.(2.10). But we will see that some information can be obtained anyway.

The last equation is solved by trigonometric functions, except for $n = 0, \pm 1$. We note that $|n| = 1$ are zero-modes, and $n = 0$ just gives the expected exponential divergence $\delta\theta \sim e^\tau$, which is an artifact of taking $\theta = \pi/2$ in the equations, c.f. the simple plane pendulum in the vertical upright position.

Let us now look at the first 4 equations and apply the method of the previous case

$$\begin{pmatrix} \dot{\phi}_n^1 \\ \dot{p}_n^1 \\ \dot{\phi}_n^2 \\ \dot{p}_n^2 \\ \dot{\phi}_n^3 \\ \dot{p}_n^3 \\ \dot{\phi}_n^4 \\ \dot{p}_n^4 \end{pmatrix} + \mathcal{A} \begin{pmatrix} \phi_n^1 \\ p_n^1 \\ \phi_n^2 \\ p_n^2 \\ \phi_n^3 \\ p_n^3 \\ \phi_n^4 \\ p_n^4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (5.14)$$

where the matrix \mathcal{A} is

$$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ n^2 & 0 & -2\kappa in & 0 & 0 & -2^{3/2}\kappa & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 2\kappa in & 0 & n^2 + 4(\kappa^2 - 1) & 0 & 0 & 0 & 0 & 2^{3/2} \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 2^{3/2}\kappa & 0 & 0 & n^2 - 2(\kappa^2 - 1) & 0 & 2in & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -2^{3/2} & -2in & 0 & n^2 & 0 \end{pmatrix} \quad (5.15)$$

Again we want to find the condition for which all the eigenvalues λ are imaginary. For $\lambda^2 = l$ the characteristic polyomium is

$$\begin{aligned} p(l) = & l^4 + l^3(4n^2 + 10\kappa^2 + 6) + l^2(6n^4 + 24\kappa^4 + 32\kappa^2 + 18\kappa^2n^2 + 6n^2 + 8) \\ & + l(4n^6 - 6n^4 + 16n^2 + 6n^4\kappa^2 + 24\kappa^4n^2 + 24\kappa^2n^2) \\ & + n^8 - 6n^6 + 8n^4 - 2n^6\kappa^2 + 8n^4\kappa^2 \end{aligned} \quad (5.16)$$

We first show that the 3 extrema fall at negative l , i.e. that the roots of

$$\begin{aligned} p'(l) = & 4l^3 + 3l^2(4n^2 + 10\kappa^2 + 6) \\ & + 2l(6n^4 + 24\kappa^4 + 32\kappa^2 + 18\kappa^2n^2 + 6n^2 + 8) \\ & + (4n^6 - 6n^4 + 16n^2 + 6n^4\kappa^2 + 24\kappa^4n^2 + 24\kappa^2n^2) \end{aligned} \quad (5.17)$$

are negative. For that purpose, it is enough to show that the 2 extrema of $p'(0)$ fall at negative l , and that $p'(0) > 0$. We get

$$p''(l) = 12l^2 + 6l(4n^2 + 10\kappa^2 + 6) + 2(6n^4 + 24\kappa^4 + 32\kappa^2 + 18\kappa^2n^2 + 6n^2 + 8) = 0 \quad (5.18)$$

It is easily seen that both of the solutions of this equation fulfill the requirement for arbitrary n, κ . We then have to look at $p'(0)$

$$p'(0) = 4n^6 - 6n^4 + 16n^2 + 6n^4\kappa^2 + 24\kappa^4n^2 + 24\kappa^2n^2 \quad (5.19)$$

Given that the last three terms are positive, and that the first three can be written as

$$4n^6 - 6n^4 + 16n^2 = n^2(2n^2 - 2)^2 + 2n^4 + 12n^2 \quad (5.20)$$

it is seen that $p'(0) > 0$. Now we just have to find the condition that $p(0) \geq 0$, which leads to

$$(4 - n^2)(2 + 2\kappa^2 - n^2) \geq 0 \quad (5.21)$$

For $n = 0, 1, 2$, κ is arbitrary. For $n = 3, 4, 5, \dots$ we get $\kappa \leq \sqrt{\frac{7}{2}}, \sqrt{7}, \sqrt{\frac{23}{2}}, \dots$, respectively. Thus we end up with $\kappa^2 \leq 7/2$, corresponding to the spin

$$S \leq \frac{15}{8\sqrt{2}H^2\alpha'} \quad (5.22)$$

indicating, that we have better stability as compared to the $A = 0$ case.

There is actually another case where we can solve the equations analytically, namely when $\kappa = A$. As seen from (2.10) and (2.12), this limit corresponds to zero spin, $S = 0$. We get the equations for $i = 1, 2, 3, 4$

$$\ddot{\phi}_n^1 + n^2\phi_n^1 - \sqrt{4(1+A^2)}\dot{\phi}_n^3 - 2in\phi_n^2 = 0 \quad (5.23)$$

$$\ddot{\phi}_n^2 + n^2\phi_n^2 + \sqrt{4(1+A^2)}\dot{\phi}_n^4 + 2in\phi_n^1 = 0 \quad (5.24)$$

$$\ddot{\phi}_n^3 + n^2\phi_n^3 + \sqrt{4(1+A^2)}\dot{\phi}_n^1 + 2in\phi_n^4 = 0 \quad (5.25)$$

$$\ddot{\phi}_n^4 + n^2\phi_n^4 - \sqrt{4(1+A^2)}\dot{\phi}_n^2 - 2in\phi_n^3 = 0 \quad (5.26)$$

And for $i = 5$

$$\ddot{\phi}_n^5 + n^2\phi_n^5 - \frac{2 - A^2\text{sn}^2(\tau|A^2)}{\text{sn}^2(\tau|A^2)}\phi_n^5 = 0, \quad A \leq 1 \quad (5.27)$$

$$\ddot{\phi}_n^5 + n^2\phi_n^5 - \frac{A^2(2 - \text{sn}^2(A\tau|1/A^2))}{\text{sn}^2(A\tau|1/A^2)}\phi_n^5 = 0, \quad A \geq 1 \quad (5.28)$$

The first four equations can be shown to lead to oscillating solutions only, (and zero-modes) by the same method as used before, so we skip the details.

Since the term in front of ϕ_n^5 is periodically minus infinity, the equations for ϕ_n^5 lead to instabilities. It is not surprising that the pulsating strings become classically unstable for $S \approx 0$. Notice that the length of the unperturbed string is

$$L(\tau) = \int_0^{2\pi} \frac{ds}{d\sigma} d\sigma = \frac{2\pi}{H} \sqrt{H^2 r_0^2 + \sin^2 \theta(\tau)} \quad (5.29)$$

$S \approx 0$ corresponds to $r_0 \approx 0$, which gives $L(\tau) = 2\pi H^{-1} |\sin \theta(\tau)|$. It follows that the string oscillates between its maximal size and zero size. It is well-known that the classical perturbations blow up when a circular string collapses to a point. But, of course, the classical approximation cannot be trusted in this limit.

6 Numerical Analysis

Given the equations as they stand, eqs.(4.13)-(4.17), we have 5 coupled second order ordinary differential equations for the Fourier components ϕ_n^i .

Needless to say, except for the few special cases considered in the previous section, any attempts at solving the equations analytically end up being an exercise in futility. The correct way of going about the problem is to solve the equations numerically. In what follows, we first convert the 5 second order ordinary differential equations to 10 first order ordinary differential equations, and then solve them numerically, using an appropriate algorithm. A good safe choice is the classic fourth order Runge Kutta method for step size h .

In order to run this algorithm, we must provide the step size h , and the total number of steps, N . In addition to that, we must also supply boundary conditions. The final data was analysed by printing the 10 graphs for the 5 complex solutions, $\phi_n^1 - \phi_n^5$.

All the aforementioned issues have been dealt with on a trial and error basis. The step size has been made gradually smaller, until there were no noticeable changes in the output, i.e., no changes on the fifth decimal place. In terms of N , we looked at the graphs and determined, that roughly after 100000 points, it was clear whether the functions diverged or not.

Ideally, the stability of the equations should be tested for an infinite number of possible boundary conditions, but since that is impossible, we have had to content ourselves with being able to only check a limited number of boundary conditions for the solutions. We have not made a major issue out

of it, and have only varied the boundary conditions for a couple of stable and unstable solutions. This minor analysis showed, in terms of the behaviour of the functions for large τ , total insensitivity to varying the boundary conditions. Given that we are dealing with complex functions, we have chosen finite different complex boundary conditions around unity for all the functions.

We now turn to the numerical results. For each (A, n) we integrate the 5 second order equations for $Hc_0 = \kappa > A$, to determine whether we have stability or not. Typically we have stability up to a critical κ , beyond which we have instability. An example is shown in Figures 1 and 2, for $(A, n) = (2, 3)$. In this case we have stability up to $\kappa \approx 2.54$, and instability from $\kappa \approx 2.56$, which in terms of the spin is

$$S \leq 1.71(H^2\alpha')^{-1}, \quad A = 2 \quad (6.1)$$

To find the critical κ which separates the stable and unstable solutions, we now let A and n run. The corresponding critical spin S , obtained from (2.12), is plotted as a function of A in Figure 3.

For $A = 0, 1$ we found, analytically, that the modes $n = 0, 1, 2$ are either zero-modes or stable. This turns out to hold for arbitrary A . We also saw in the same cases that $n = 3$ was the most unstable mode. This also turns out to be the case for arbitrary A ; see again Figure 3.

More precisely, in Figure 3, we show the maximal spin for which we have stability as a function of A . We only show the curves for the modes $n = 3, 4, 5$, just to show that $n = 3$ is the most unstable mode, as already mentioned.

The numerical results are of course in agreement with the analytical results for $A = 0, 1$, and from Figure 3 we can then finally conclude that the inclusion of pulsation leads to better stability. In other words, the more pulsation we have (the higher A), the higher spin we can allow and still have stability.

7 Concluding Remarks

In conclusion, we have shown that when we include pulsation in S^5 [26], the resulting string solution has better stability properties than the original two-spin string solution in AdS_5 [30]. It would be interesting to see if pulsation could also improve the stability properties of the multi R-charge solutions in S^5 [30]. Unfortunately, however, the equations for the perturbations become even more complicated than our eqs.(4.13)-(4.20), meaning that the analysis will demand extensive numerical work.

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A Appendix

The second fundamental form and normal fundamental form give (we only show the non-zero components)

$$K_{\tau\tau}^3 = K_{\sigma\sigma}^3 = -r_0\sqrt{1 + H^2r_0^2} \quad (\text{A.1})$$

$$K_{\tau\tau}^5 = K_{\sigma\sigma}^5 = \frac{-\sqrt{c_0^2 - r_0^2} \sin \theta \cos \theta}{\sqrt{H^2r_0^2 + \sin^2 \theta}} \quad (\text{A.2})$$

$$K_{\tau\sigma}^2 = \omega r_0 \quad (\text{A.3})$$

$$K_{\tau\sigma}^4 = \frac{-r_0 \cos \theta \dot{\theta}}{\sqrt{H^2r_0^2 + \sin^2 \theta}} \quad (\text{A.4})$$

$$\mu_{\sigma}^{21} = -\mu_{\sigma}^{12} = \frac{-c_0\sqrt{1 + H^2r_0^2}}{\sqrt{c_0^2 - r_0^2}} \quad (\text{A.5})$$

$$\mu_{\sigma}^{25} = -\mu_{\sigma}^{52} = \frac{r_0\omega\dot{\theta}}{\sqrt{c_0^2 - r_0^2}\sqrt{H^2r_0^2 + \sin^2 \theta}} \quad (\text{A.6})$$

$$\mu_{\sigma}^{34} = -\mu_{\sigma}^{43} = \frac{-\sqrt{1 + H^2r_0^2} \sin \theta}{\sqrt{H^2r_0^2 + \sin^2 \theta}} \quad (\text{A.7})$$

$$\mu_{\sigma}^{45} = -\mu_{\sigma}^{54} = \frac{-H^2r_0 \cos \theta \sqrt{c_0^2 - r_0^2}}{H^2r_0^2 + \sin^2 \theta} \quad (\text{A.8})$$

$$\mu_{\tau}^{31} = -\mu_{\tau}^{13} = \frac{c_0\omega}{\sqrt{c_0^2 - r_0^2}} \quad (\text{A.9})$$

$$\mu_{\tau}^{42} = -\mu_{\tau}^{24} = \frac{-\omega \sin \theta}{\sqrt{H^2r_0^2 + \sin^2 \theta}} \quad (\text{A.10})$$

$$\mu_{\tau}^{53} = -\mu_{\tau}^{35} = \frac{r_0\dot{\theta}\sqrt{1 + H^2r_0^2}}{\sqrt{c_0^2 - r_0^2}\sqrt{H^2r_0^2 + \sin^2 \theta}} \quad (\text{A.11})$$

Now the Riemann component terms

$$R_{\mu\rho\nu\sigma}N_i^{\mu}N_j^{\nu}(X'^{\rho}X'^{\sigma} - \dot{X}^{\rho}\dot{X}^{\sigma}) \quad (\text{A.12})$$

lead to

$$R_{\mu\rho\nu\sigma}N_1^\mu N_1^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = -H^2 c_0^2 \quad (\text{A.13})$$

$$R_{\mu\rho\nu\sigma}N_2^\mu N_2^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = -H^2 c_0^2 \quad (\text{A.14})$$

$$R_{\mu\rho\nu\sigma}N_3^\mu N_3^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = -H^2 c_0^2 \quad (\text{A.15})$$

$$R_{\mu\rho\nu\sigma}N_4^\mu N_4^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = H^2(2r_0^2 - c_0^2) \quad (\text{A.16})$$

$$R_{\mu\rho\nu\sigma}N_5^\mu N_5^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = -H^2(2r_0^2 - c_0^2) \quad (\text{A.17})$$

$$R_{\mu\rho\nu\sigma}N_6^\mu N_6^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = 2\sin^2\theta - H^2(c_0^2 - 2r_0^2) \quad (\text{A.18})$$

$$R_{\mu\rho\nu\sigma}N_7^\mu N_7^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = 2\sin^2\theta - H^2(c_0^2 - 2r_0^2) \quad (\text{A.19})$$

$$R_{\mu\rho\nu\sigma}N_8^\mu N_8^\nu(X'^\rho X'^\sigma - \dot{X}^\rho \dot{X}^\sigma) = 2\sin^2\theta - H^2(c_0^2 - 2r_0^2) \quad (\text{A.20})$$

References

- [1] H.J. de Vega, A.L. Larsen and N. Sánchez, Phys. Rev. **D51**, 6917 (1995).
- [2] H.J. de Vega and I.L. Egusquiza, Phys. Rev. **D54**, 7513 (1996).
- [3] J. Maldacena, Adv. Math. Phys. **2**, 231 (1998), Int. J. Theor. Phys. **38**, 1113 (1999).
- [4] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. **B428**, 105 (1998).
- [5] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).
- [6] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Nucl. Phys. **B636**, 99 (2002).
- [7] D.J. Gross and F. Wilczek, Phys. Rev. **D9**, 980 (1974).
- [8] H. Georgi and D. Politzer, Phys. Rev. **D9**, 416 (1974).
- [9] E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. **B129**, 66 (1977).

- [10] G.P. Korchemsky, Mod. Phys. Lett. **A4**, 1257 (1989).
- [11] F.A. Dolan and H. Osborn, Nucl. Phys. **B629**, 3 (2002).
- [12] S. Frolov and A.A. Tseytlin, JHEP **0206**, 007 (2002).
- [13] A.A. Tseytlin, Int. J. Mod. Phys. **A18**, 981 (2003).
- [14] J.G. Russo, JHEP **0206**, 038 (2002).
- [15] A. Armoni, J.L.F. Barbon and A.C. Petkou, JHEP **0206**, 058 (2002).
- [16] G. Mandal, Phys. Lett. **B543**, 81 (2002).
- [17] J.A. Minahan, Nucl. Phys. **B648**, 203 (2003).
- [18] A. Armoni, J.L.F. Barbon and A.C. Petkou, JHEP **0210**, 069 (2002).
- [19] M. Axenides and E. Floratos, Nucl. Phys. **B662**, 170 (2003).
- [20] M. Alishahiha and A.E. Mosaffa, JHEP **0210**, 060 (2002).
- [21] A. Buchel, Phys. Rev. **D67**, 066004 (2003).
- [22] R.C. Rashkov and K.S. Viswanathan, “Rotating Strings with B-Field”,
hep-th/0211197 (unpublished).
- [23] S. Ryang, JHEP **0304**, 045 (2003).
- [24] D. Aleksandrova and P. Bozhilov, JHEP **0308**, 018 (2003).
- [25] D. Aleksandrova and P. Bozhilov, Int. J. Mod. Phys. **A19** 4475 (2004).
- [26] A. Khan and A.L. Larsen, Phys. Rev. **D69**, 026001 (2004).
- [27] M. Smedback, JHEP **0407**, 004 (2004).
- [28] F. Bigazzi, A.L. Cotrone and L. Martucci, Nucl. Phys. **B694**, 3 (2004).

- [29] F. Bigazzi, A.L. Cotrone, L. Martucci and L.A. Pando Zayas, "Wilson Loop, Regge Trajectory and Hadron Masses in a Yang-Mills Theory from Semiclassical Strings", hep-th/0409205 (unpublished).
- [30] S. Frolov and A.A. Tseytlin, Nucl. Phys. **B668**, 77 (2003).
- [31] J. Engquist, J.A. Minahan and K. Zarembo, JHEP **0311**, 063 (2003).
- [32] N. Beisert, J.A. Minahan, M. Staudacher and K. Zarembo, JHEP **0309**, 010 (2003).
- [33] S. Frolov and A.A. Tseytlin, JHEP **0307**, 016 (2003).
- [34] S.A. Frolov, I.Y. Park and A.A. Tseytlin, Phys. Rev. **D71**, 026006 (2005).
- [35] S. Frolov and A.A. Tseytlin, Phys. Lett. **B570**, 96 (2003).
- [36] G. Arutyunov, S. Frolov, J. Russo and A.A. Tseytlin, Nucl. Phys. **B671**, 3 (2003).
- [37] N. Beisert, S. Frolov, M. Staudacher and A.A. Tseytlin, JHEP **0310**, 037 (2003).
- [38] G. Arutyunov, J. Russo and A.A. Tseytlin, Phys. Rev. **D69**, 086009 (2004).
- [39] B. Stefanski, JHEP **0403**, 057 (2004).
- [40] N. Kim, Phys. Rev. **D69**, 126002 (2004).
- [41] A.A. Tseytlin, "Spinning Strings and AdS/CFT Duality", hep-th/0311139 (unpublished).
- [42] A.L. Larsen and A. Khan, Nucl. Phys. **B686**, 75 (2004).
- [43] M. Alishahiha, A.E. Mosaffa, H. Yavartanoo, Nucl. Phys. **B686**, 53 (2004).

- [44] H. Dimov and R.C. Rashkov, JHEP **0405**, 068 (2004).
- [45] B. Stefanski and A.A. Tseytlin, JHEP **0405**, 042 (2004).
- [46] J. Engquist, JHEP **0404**, 002 (2004).
- [47] K. Ideguchi, JHEP **0409**, 008 (2004).
- [48] S. Ryang, JHEP **0410**, 059 (2004).
- [49] S. Bellucci, P.Y. Casteill, J.F. Morales and C. Sochichiu, Nucl. Phys. **B707**, 303 (2005).
- [50] S. Ryang, JHEP **0404**, 053 (2004).
- [51] M. Kruczenski, A.V. Ryzhov and A.A. Tseytlin, Nucl. Phys. **B692**, 3 (2004).
- [52] H. Ebrahim and A.E. Mosaffa, JHEP **0501**, 050 (2005).
- [53] H.J. de Vega and N. Sánchez, Nucl. Phys. **B317**, 706 (1989).
- [54] H.J. de Vega and N. Sánchez, Nucl. Phys. **B317**, 731 (1989).
- [55] H.J. de Vega and N. Sánchez, Phys. Lett. **B244**, 215 (1990).
- [56] A.L. Larsen and V.P. Frolov, Nucl. Phys. **B414**, 129 (1994).
- [57] J. Guven, Phys. Rev. **D48**, 4604 (1993).
- [58] J. Guven, Phys. Rev. **D48**, 5562 (1993).
- [59] S. Kar, Phys. Rev. **D54**, 6408 (1996).

Figure Captions

Figure 1. Example of a stable solution for $A = 2$, $\kappa = 2.54$, $n = 3$. All perturbations are finite oscillating functions.

Figure 2. Example of an unstable solution for $A = 2$, $\kappa = 2.56$, $n = 3$. Some perturbations blow up.

Figure 3. Maximal spin S (in units of $(H^2\alpha')^{-1}$), for which we have stable solutions, as a function of A .

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